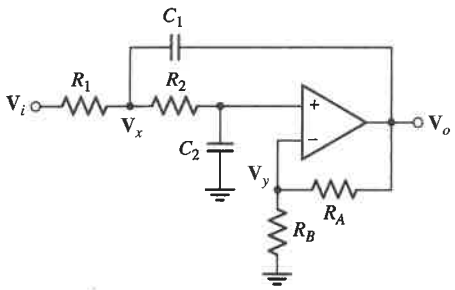
TABLE **16.2** Coefficients for Low-Pass Butterworth and Chebyshev ($\beta = 0.9976$, or 3 dB) Filter Functions, Normalized to $\omega_c = 1$

Butterworth

n	a_0	a_1	a_2	a_3	a_4
1	1.0000				
2	1.0000	1.4142			
3	1.0000	2.0000	2.0000		
4	1.0000	2.6131	3.4142	2.6131	
5	1.0000	3.2361	5.2361	5.2361	3.2361

Chebyshev ($\beta = 0.9976$)

n	a_0	a_1	a_2	a_3	a_4
1	1.0024				
2	0.7080	0.6449			
3	0.2506	0.9284	0.5972		
4	0.1770	0.4048	1.1691	0.5816	
5	0.0626	0.4080	0.5489	1.4150	0.5744



■ **FIGURE 16.48** General low-pass Sallen-Key filter circuit.

If we wish to improve upon this basic approach, a circuit worth considering is known as the Sallen-Key amplifier, shown in Fig. 16.48, configured as a low-pass filter. Analysis of this circuit by nodal analysis is straightforward. We first define the gain G of the noninverting amplifier as

$$G \equiv \frac{R_A + R_B}{R_B} \qquad [37]$$

Then voltage division yields

$$\mathbf{V}_y = \mathbf{V}_x \frac{1}{1 + R_2 C_2 \mathbf{s}} \qquad [38]$$

and we may write a single nodal equation

$$0 = \frac{\mathbf{V}_x - \mathbf{V}_i}{R_1} + \frac{\mathbf{V}_x - \mathbf{V}_y}{R_2} + \frac{\mathbf{V}_x - \mathbf{V}_o}{1/\mathbf{s}C_1} \qquad [39]$$

Subtituting Eqs. [37] and [38] into Eq. [39] and performing a few algebraic maneuvers, we arrive at an expression for the transfer function of the amplifier,

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{G/R_1 R_2 C_1 C_2}{\mathbf{s}^2 + \left[\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}\right]\mathbf{s} + \frac{1}{R_1 R_2 C_1 C_2}} \qquad [40]$$

Noting that the coefficients in Table 16.2 represent filters with a cutoff frequency of 1 rad/s, so that when finished we will need to make use of the simple scaling techniques described in Sec. 16.5, we are now ready to explore the design of a second-order Butterworth low-pass filter.